The Mathematics of Magic: The Gathering

A study in probability, statistics, strategy, and game theory

By Jon Prywes

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Here are some of Jon Prywes' Magic accomplishments:

He wrote a Magic magazine online called The Library of Leng, from 1995 to 1997;

He wrote three articles for Scrye Magazine in 1996 and 1997;

He started a Magic club at his high school in 1997, which ran through 1999;

He played in several semi-competitive tournaments including the 1999 Junior Super Series Eastern Divisional;

He wrote numerous articles for The Magic Dojo (featured on this page);

He wrote a paper about the mathematical components of Magic in 1999 (also featured on this page);

He has done hundreds of Magic eBay auctions;

He worked at a day camp teaching Magic strategy to kids in the summer of 1999

Table of Contents

Introduction to Magic: The Gathering

A Mathematical Introduction....3

A Description of the Game....4

A Sample Game....6

Why Experience Counts....7

Two-Person Game Theory

What Does It Mean?....9

The Basic Concepts....10

Two-Person Game Theory and Magic: The Gathering....12

Probability

Probability and Magic: The Gathering....15

Shuffling and Randomization....16

Drawing Cards From a Deck....17

The Average Game....2

Chance Versus Skill....3

Deckbuilding

Deckbuilding and Magic: The Gathering....5

Deck Archetypes and Deck Strategy....6

Deck Comparison and Winning Ratios....7

Card Efficiency and Resource Management....10

The Big Game

Probability, Statistics, Game Theory, and Magic....11

Math and the Average Player....12

Personal Applications....14

Introduction to Magic: The Gathering

A Mathematical Introduction

Magic: The Gathering is a game which some take seriously, and others lightly. Many understand it, even more do not. Some play it nonstop and others never see the point. Personally, I think it is a great game. Only those who actually play the game can truly understand how wonderful it is. Most people who write it off as pointless do not see the mechanics of the game that make it simple, yet complex. In its five years of existence, the crowd that Magic has drawn can alone show how great the game really is. On the weekend of May 1 and 2 I traveled with some friends to Pro Tour: New York in Secaucus, New Jersey. Hundreds of people were gathered inside the Meadowlands Exposition Center just to play the game they love. The Pro Tour is no day in the park; it is a great competition for the mind.

Zev Gurwitz, a junior at Scarsdale High School, along with myself, had both qualified for the Junior Super Series event. Sixty-four players age seventeen and under qualify for the JSS at challenge tournaments held during the winter. We had playtested for this tournament a fairly lengthy amount, yet I was still skeptical. As it turned out, I went 1-3 and dropped out. Zev went 0-3 and dropped. A record of at least four wins was necessary to make the top eight, in order to play at the JSS Championships where $250,000 in scholarship money is awarded. We tried, but what was it that kept us from winning? What is it that keeps every Magic player from winning all the time? What keeps even the most skilled of players at bay from winning every single game? It is a word called chance.

Chance is what makes Magic different from many other classic games. When one plays Chess, he gets to see all the pieces. Both players have eight pawns and eight other pieces, either white or black. Every piece moved is public information, and nothing is held secret. Chess is ruled by complete skill. Nothing random can possibly happen. Magic, on the other hand, adds a new element. Not only are each person's playing materials often different or varied (the decks), but randomness exists in this game. One starts the game by randomizing his deck of sixty or more cards, then drawing seven. He will definitely not draw the same seven cards in every opening hand! Variety is what makes this game different than Chess. There are two elements of variety involved: Deck construction and random card draws.

Deck construction is part of what governs this element of variety. The card draws are simply a result of the deck construction, as well as any cards played during the game which may alter the contents of what is remaining in the deck (for example, removing cards from the deck). The Duelists' Convocation International (DCI) is the authority governing sanctioned tournaments. Although many players do not participate in tournaments, the DCI's deckbuilding guidelines provide for a fair game.

Some cards were printed before the research and development team realized they were overpowered. Hence, the cards are banned from deck construction. Some cards are only restricted, meaning only one is allowed in a deck. Any card that is not banned or restricted is allowed up to four copies per deck. A player has to think, "how many copies of a card do I want in my deck? One, two, three, or four?" If he puts four copies in, he is likely to draw one very soon. If he only uses one, his chances of drawing it are slim. Maybe having multiple copies is redundant and that is the reason for using fewer. If he uses three copies of a card, then what will be his chances of drawing one by turn six? Probability pokes its nose up into the game now.

In Chess, there is no probability, other than thinking about the number of possible moves. However, a player's play style is more likely to govern how he plays the game of Chess than probability will. For all intents and purposes, there is no probability in Chess. Probability goes hand in hand with chance and random events. It is the chance that an event will occur, or not occur. Will your adversary draw the card he needs to win the game this turn? Next turn? Within five turns? Calculating the chances can help a player decide whether to play defensively and anticipate the other person drawing their win card, or play aggressively and assume that the card will not be drawn by the other player.

Some of the other math involved in the game is less obvious. The "chance versus skill" question is a very hot topic among high-level players. How much of playing a given deck is based on skill, and how much on chance? The obvious solution would be to take a fairly inexperienced player and give him one deck, and give an experienced player the other.

The experienced player will win with either deck if there is complete skill involved. If the deck is straightforward to the point that there are no decisions to be made, and it beats the other deck almost automatically, then it is possible the expert will lose games. In Magic, though, there are always at least some thoughtful decisions to make. The chance of getting paired against a superior deck in a tournament does not mean an automatic loss. If your skill is lower to that of the superior deck your chances may be slim, but if your skill is higher than your chances become much greater.

So then, what constitutes a superior deck? What deckbuilding techniques are necessary for one to make a deck that wins more often than a deck which is not as good? How does one determine which card selections will guarantee a flawless road to victory? Questions like these are what every player must try to answer when attempting to create the "perfect" deck. Of course, I will aim to prove that the "perfect" deck does not exist. Laws of game theory are an invaluable aid in proving this. While game theory is a very conceptual science, its laws very well do apply to games such as Magic. While knowledge of these laws will not make one a Pro Tour Player, they can help one who is trying to understand the game have an easier time making decisions. Game theory is simply a series of laws regulating how one goes about making decisions in a situation. A "game" in game theory is not necessarily always a game as defined in common talk, but game theory certainly has many game applications.

Through the laws of probability and game theory, along with statistical analysis and actual experiments, I will be working on coming up with conclusions that can take these laws and correlate them to the game of Magic. First and foremost, I will be explaining the basic concepts of game, in order for the reader who is unacquainted with Magic to familiarize himself with these terms. In describing the game, I will attempt not to reiterate the entire rulebook, though I will summarize the basics of how the game is played. Upon completion of these details, I will begin my outline of game theory and begin on relating the math and game theory to its Magic applications. I will provide profiles of many of the people who I consult with as part of my project. I will be discussing a survey I conducted in order to determine how much math Magic players recognize as part of the game. This entire report will follow closely the outline given in my project description.

A Description of the Game

The object of Magic is simple. You start with 20 life points, and when they are reduced to 0, the game is over and the other person wins. You and another person both have a deck of sixty or more Magic cards, which are used in playing the game. The cards can be from a variety of different sets, with (for the most part) no more than four of any one card. The players alternate in taking turns. Each turn is a sequence of events that involve drawing a card, putting cards into play, "attacking" the other player (using cards that represent creatures) and then discarding if necessary.

The basic resource in Magic is called mana. Mana comes from a Polynesian word meaning energy. Mana let you bring certain cards into play, and use abilities on cards already in play. Some cards require more mana to use than others do. There are five different colors of mana in the game, each representing a separate force. The forces represented are typical of an adventure gaming genre: White mana represents the powers of good. Blue mana is for the powers of the mind. Black mana is for the powers of evil. Red mana is for the powers of destruction and chaos. Green mana is for the powers of nature and wildlife. Each color's theme is represented in the cards of that color.

Thus a well-known adventure theme is presented in the cards, giving players something more than just a strategy game. It is a game with a theme, with the strategy hiding in the background for the more advanced players to take notice of. In addition, some cards do not have colors; these cards are either land (which produce mana), or artifacts (which are colorless and can be played with any kind of mana). All cards in the game are referred to as "spells" for game purposes, except for land. Lands are not spells and are not "cast". They are simply placed into play. I will often use terms interchangeably, however.

Another concept that the player must understand is the word "tap." To tap a card is to rotate it sideways, indicating that its powers have been used. This can be used to represent an attacking creature, a used artifact, or a land drawn for mana. At the beginning of each turn you untap all your cards, so you effectively can use their powers once each turn (and/or during your opponent's turn).

A spell's casting cost is the amount of mana you need to use in order to play it. It is located in the top right corner of the card. If a card's casting cost is 2R, for instance, that means that you must spend one red mana and 2 of any color to play it. Casting cost will be referred to a lot so this is very important to understand. The colored portion is specific, and the numbered portion is generic, and can be paid using any color mana.

There are different types of cards in the game: Land, Artifacts, Creatures, Enchantments, Sorceries, and Instants. All cards come in five different colors except for lands and artifacts. Lands are a special kind of card; you can play one each turn. Lands can be tapped for mana, which is used to play any of the other kinds of cards. Artifacts are colorless, which means you can use any kind of mana to play them. They may let you do anything from draw cards to affect cards in play.

Some artifacts are also creatures. Creatures can be used to do damage to your opponent (thus reducing his life total from 20 to 0). However, if your opponent has creatures of his own out, he can use them to block yours. Enchantments are cards played on an existing card, which modify what the card does, usually. An example would be an Enchant Creature card, which would be played on a creature. It might make the creature weaker, or stronger. Some enchantments are not played on other cards, and have a global effect on the game. Sorceries and instants cause a one-time effect on the game. Sorceries can only be played during your turn; instants can be played anytime.

At the beginning of the game each player shuffles his deck. The players roll a die or flip a coin to determine who chooses who goes first, then each draws seven cards. The player who chooses to go first does not draw a card on his first turn. The sequence of a turn is as follows:

Untap phase: Untap all cards you control. This means to rotate them so they are all facing upward and not rotated (tapped).

Upkeep phase: This is a maintenance phase. Some cards will make you do an effect during this phase, such as pay mana to keep the card in play, for example.

Draw phase: Draw a card.

Main phase: You can do these things, in any order:

a. Put one land into play

b. Declare one attack

c. Play spells. You can play spells before or after your attack, as well as both. Note that all cards (except for instants) can only be played on your turn during the main phase.

Discard phase: Discard down to seven cards.

Cleanup phase: Any effect that lasts until "end of turn" wears off now. Any damage on a creature, which does not destroy it, wears off as well.

The attack works like this: You choose any untapped creatures you control that you have had in play at least one turn, and tap them. Your opponent either blocks them or takes damage equal to their power. If he blocks, both creatures deal damage to each other equal to their power. A creature has a pair of numbers in the bottom right corner. These are its power and toughness. When a creature deals damage, it deals damage equal to its power. When it receives, the damage is applied to its toughness. If it takes damage equal to or greater than its toughness it will be buried. That means it will be played in a pile next to your draw pile called your discard pile. For example, a Giant Spider has a power and toughness of 2/4. It deals 2 damage to a creature blocking or blocked by it, and if unblocked during an attack, deals 2 to the player it attacked. If it takes 4 damage during one turn, it will be placed in its controller's discard pile.

The game gets a lot more complex than this with the framework of the rules, but these are the basics. What the cards do is another story. There are a wide variety of abilities in Magic, and this summary is just the foundation. To fully understand the game, one must see it being played. Otherwise the rules are hard to make sense out of. It may seem daunting at first, but after many games, it becomes automatic. The strategy is immense, and the game leaves players with many decisions to make. These decisions are what make the game the strategic task it is.

A Sample Game

Perhaps, at this point, one who is unfamiliar with the game still may be confused. A sample game can at least demonstrate the flow of the game and give a general idea. However, to one who is already familiar with Magic, this will be superfluous. I will be introducing new terms and concepts as the game progresses. In this example I will be playing a deck which is red and has many artifacts. My proverbial adversary will be using a green deck with some white cards. This is based on a game I played online using Apprentice, a program that allows the online play of Magic through a graphical user interface.

The life total score will be presented in the notation X/Y, where X is my score and Y the other person's. Cards will be footnoted in the notation: Card type, casting cost. Card text. Power/Toughness. Colored mana symbols will be represented by B for black, U for blue, G for green, R for red, and W for white. At the beginning of the game, both players shuffle their decks and draw seven cards. I will win coin flip and decide to go first.

Turn 1 (Me): Skip first draw because of play-draw rule (whoever goes first skips his first draw phase to cancel out the advantage of getting to play first). Play Mountain(1). Tap Mountain to play Voltaic Key(2). Declare end of turn. 5 cards in hand. (20/20)

Turn 1 (Opp): Draw a card. Play Forest(3). Tap Forest to play Elvish Lyrist(4). Declare end of turn. 6 cards in hand. (20/20)

Turn 2 (Me): Untap Mountain. Draw a card. Play Mountain. Tap two Mountains to play Grim Monolith(5). Playing this card in conjunction with the Voltaic Key allows for enhanced mana production, as the Key can be used to untap the Monolith. Declare end of turn. 4 cards in hand. (20/20)

Turn 2 (Opp): Untap Forest. Draw a card. Play Brushland(6). Tap Forest and Brushland to play Priest of Titania(7). Attack with Elvish Lyrist for 1 damage. Declare end of turn. 5 cards in hand. (19/20)

Turn 3 (Me): Untap. (Untap all cards which don't say otherwise, such as Grim Monolith which says it does not untap. However, it can be untapped in other methods. For instance, it says you can pay 4 mana at any time to untap it. The Voltaic Key allows you to pay a mana and tap it to untap any artifact in play, so that, too, can be used to untap the Monolith. However, it is not tapped right now so that is not a problem.) Draw a card. Play Mountain. Tap 3 mountains to cast Stone Rain(8) on opponent's Forest. For 2 mana of any color and one that red, this will make my opponent lose a land. This can be beneficial because by denying the opponent mana, he cannot play as many cards. 3 cards in hand. Declare end of turn. (19/20)

Turn 3 (Opp): Untap. Draw a card. Play another Forest. Attack with Elvish Lyrist, and during the attack, tap the Forest for a green mana, and the Brushland for a generic mana. The Priest of Titania will yield 2 green mana, since there are 2 Elves in play, for a total of 4 mana. He then plays Might of Oaks(9) on the attacking Lyrist. Dealing 7 extra damage for 4 mana is a very large benefit. Later on I will be analyzing card utility, but for now I will just stick to the basics. Making the creature an 8/8 until end of turn, it deals 8 damage to me. 4 cards in hand. (11/20)

Turn 4 (Me): Untap. Draw a card. Right now I am worried that my opponent has more Might of Oaks at his disposal so I want to make sure he cannot use them. I know that there can be anywhere from 0 to 3 more left in his deck, and since he has taken 3 turns he has exhausted 10 cards from his deck. With 50 cards left and 4 in his hand, what are the chances he will have one in his hand after drawing a card next turn? This is yet another question I will be exploring. For now, however, assume I am mathematically unintelligent and do not know. My main goal will be to get rid of his creatures so

I can stop taking damage. I tap the Monolith for 3 mana. I use one with the Key to untap the Monolith, and tap it again. I tap all my Mountains. I now have 9 mana, 4 of which is red. I use 1 red and 3 generic to play Avalanche Riders(10). I destroy my opponent's lone Forest again. I then play Wasteland(11). I decide I want to destroy my opponent's Brushland with my Wasteland, leaving him with no land.

Doing the math, I now have 5 mana remaining, 3 of which are red. I then cast Rolling Thunder(12). Since it has X, I decide to pay 2 for X with two generic mana. I do one damage to the Elvish Lyrist and one the Priest of Titania. My opponent no longer has any cards in play, and I use the remaining mana to play a Cursed Scroll (an artifact that can tap to deal damage). I attack with the Riders, dealing 2 damage. I am at a lower life then my opponent, and have fewer cards in hand; however, I am in a position to win this game right now. The game may not be technically over yet, but there is little my opponent can do to compensate for the advantage I have gained. (11/18)

I am sure the nontechnical reader is confused at this point about what has been going on. All these different game mechanics may seem awfully confusing. However, this is just a sampling of how complex this game is. Yet after playing many games it becomes pretty basic. The flow of the game is basically untap, draw, attack, play cards, and say you are done. With a few different concepts added here and there, that is how the game is played. These different concepts and card mechanics add the element of decision making to the game. The process of making these decisions is what contributes to the role of mathematical reasoning and strategy behind the cards.

Footnotes:
Mountain: Land. Tap for one red mana.
Voltaic Key: Artifact, 1. 1, Tap: Untap target artifact.
Forest: Land. Tap for one green mana.
Elvish Lyrist: Creature – Elf, G. G, Tap: Sacrifice Elvish Lyrist to Destroy target enchantment. 1/1. (Sacrificing a card simply means placing it in your discard pile. The term simply implies that it is voluntary).
Grim Monolith: Artifact, 2. Tap for 3 generic mana. Does not untap normally, and costs 4 to untap any other time. (Generic mana does not have color; they may only be used to play cards or part of cards not requiring colored mana. Note that colored mana is downwardly compatible; it may be used to play artifacts and such).
Brushland: Land. Tap for one green or white mana and take a point of damage, or tap for a generic mana.
Priest of Titania: Creature – Elf, G. Tap for a green mana for every Elf in play. 1/1.
Stone Rain: Sorcery, 2R. Destroy target land.
Might of Oaks: Instant, 3G. Target creature gets +7/+7 until end of turn.)
Avalanche Riders: Creature. Pay the casting cost during your next upkeep or bury this card (referred to as "echo"). When Avalanche Riders comes into play, destroy target land. Avalanche Riders can attack the turn it comes into play. 2/2.
Wasteland: Land. Tap for one colorless mana, or tap and sacrifice to destroy a nonbasic land. (Nonbasic means it is not one of the five basic land types: Forest, Mountain, Island, Swamp, and Plains. Basic lands are the only cards not limited to four per deck.)
Rolling Thunder: Sorcery, XRR. Divide X damage among any number of creatures and/or players.

Why Experience Counts

How much of a role in the game does experience make? What is it that prevents someone who just picked up a deck of Magic cards the other day from being a better player than someone who is a veteran at the game is? What are the parts of the game that require so much learning? The answers to this question can be studied by watching players of all different levels of experience play the game. I often play the game with players of widely varying experience. While playing, I see the factors that transcend the experienced player from the inexperienced player.

I spent the greater part of Sunday, May 16 in Long Island with my cousin and four of his friends. Being around the age of twelve, they are fairly new to the game and have not been playing as long as some of the older people with whom I play with more regularly. Jeremy Korsh, my cousin, is twelve years old and started playing the game around last spring.

Having played around one year, he has begun to pick up some of the basic concepts behind the game. However, there are many things that he has not yet picked up.

The inexperienced player is often hasty and does not wait until the right time to use the resources he has available. For instance, a Lightning Bolt(1) drawn by this kind of player will be immediately used. If his opponent has no creatures it will be aimed at the opponent, taking away 3 life points. The more experienced player would save this card for later on when he may need it more. Even still, if the opponent had a creature in play, perhaps there may be a better creature to save the Bolt for. Being able to use resources properly is a key part of the game.

Being able to recognize what makes a card playable is another skill that is notable of an experienced player. There are many cards in the game which newer players see as "godly" while experienced players will see are not effective. Such cards may include large creatures. Creatures which are as big as 9/9 or 10/10 appeal to the newer player, because of the large size. However, creature cards like these require lots of resources to bring out, and the experienced player is sure to have a card in his deck to deal with this on the spot. Counterspell(2) is an instant which lets you pay two mana to negate the effect of any card being played (except a land, because lands do not count as spells).

The inexperienced player uses all of his resources into bringing one of these large creatures into play, only to have it countered. When a spell is countered it is placed directly into the discard pile. There are several variations of the Counterspell card, which are very common-use blue cards because they let the player control the game by denying the opponent spells.

Proper uses of spells that counter other spells are another mark of experience. Cards like Counterspell, Mana Leak(3), Forbid(4), Dismiss(5) and other cards like those require some skill. Deciding which spells to counter can be very crucial to the outcome of the game. A very inexperienced player will use countermagic at the first opportunity. A slightly more experienced player will wait but still often use it on the wrong opposing spell. The experienced player knows what spells are opposing threats, and can control the game well. Learning which spells are threats comes with time. There is no given formula which can determine which spells a player should counter; only within the game can situations arise which the player has choices to make within the situation. How a player reacts to these situations is what determines his level of experience.

Perhaps my most worthy opponent (and adversary) is Zev Gurwitz. Currently a junior, he has been playing for about the same amount of time as myself. Four years of experience have taught him very good deckbuilding and playing skills. Here is where the example of proper use of countering spells comes in handy. When playing against Zev, he will almost always have a card to deal with an opposing threat. He knows to save his Counterspell or Forbid for the biggest threats.

It will always seem that Zev has countermagic because when I am hoping he does not, he will, simply because he knows what to save it for. If I have a key card in my deck, he will wait until I play it before using his countermagic. How he determines what are the biggest threats is another complexity. First, if he knows what cards his opponent has in his deck, he can know what to save his countermagic for. He will have other ways of dealing with the smaller, lesser threats. Being able to deal with an opposing situation and presenting an effective opposition yourself is what lets you win the game. Zev is pretty good at this.

Both playing and deckbuilding skills both develop over time. Some players are better players than deckbuilders, and vice versa. Some people have different playing styles. All of these factors are what comprise the many aspects of many different Magic players. Each player has individual characteristics. While one may be able to build a great deck, he may not be able to play it to its full capacity. He may stumble and make wrong decisions within the game. This could mean he may counter the wrong spells, use removal spells such as Swords to Plowshares(6) inefficiently, or use the wrong mana in playing a spell. Using a Sword on the first creature an opponent plays may be the wrong move. Or rather, it may be the right move in a given situation. A player needs to determine this. Perhaps the creature played was a Birds of Paradise(7). By using the Swords on this opposing card, the opponent's mana production will be slowed down.

Conversely, the opponent might have better targets for the Swords. The player has to decide a few things. Will I be able to deal with any other creatures if I waste this removal on the Birds? Will the advantage I gain by removing the Birds now

be worth my using this spell right now? The player has to decide whether it is an effective strategy based on the resources in his hand and deck and the resources he thinks his opponent has. Questions such as these will help him come to a conclusion about whether to play the card now or wait until later. Simply put, to play one's cards right is the way to win. Making the right decision is often easier said than done, though. Even the best of players face tough decisions. Is there always a best strategy in this game? In answering a question such as that, one can only turn to the logic of game theory.

Footnotes:
Lightning Bolt: Instant, R. Target player or creature takes 3 damage.
Counterspell: Instant, UU. Counter target spell.
Mana Leak. Instant, 1U. Counter target spell unless its caster pays 3 mana.
Forbid. Instant, 1UU. Counter target spell. You can discard two cards to put Forbid back into your hand while playing it (referred to as "buyback").
Dismiss: Instant, 2UU. Counter target spell and draw a card.
Swords to Plowshares: Instant, W. Remove target creature from the game and its controller gains life equal to its power.
Birds of Paradise: Creature, G. Tap for one mana of any color. 0/1, Flying (cannot be blocked except by creatures with flying).

Two-Person Game Theory:

What Does It Mean?

Game theory is a branch of mathematics which has been explored fairly recently within the century. It is not completely a mathematical science, however. Instead, it dictates what factors comprise strategies. It is to games of strategy what probability is to games of chance. Consequently, a game such as Magic, which relies on both strategy and chance, has both game theory and probability applications.

Most often instead of determining the best possible strategy, game theory only exists to determine the existence of a best possible strategy. Most games are too complex to be charted to the point where a best possible strategy can be determined. That is what makes game theory a mostly theoretical area of study. Nevertheless, the ideas presented in game theory are useful in outlining what the best decision making techniques are in certain situations.

There are several branches and classifications of game theory. A "game" can be one player, two players, or N-players, where N is a positive integer greater than two. There are games of perfect information where all game data is presented to all players, such as in chess, tic-tac-toe, and Monopoly. Then there are games of imperfect information where each player does not get to see all the game data. Magic and poker fall into this category, as there is an element of chance. However, there is not always chance in a game of imperfect information. For instance, take Stratego, a popular board game in which each of the two players arrange forty pieces on their side of an eight-by-eight grid, with each piece's rank facing only himself. Each player does not see the other player's pieces' ranks. Because this information is withheld from the other player, Stratego is a two-player game of imperfect information.

Additionally, a game such as Stratego is classified is a zero-sum game. A zero-sum game is one where all payoffs add up to zero. In this instance, a loss is simply a negative win, which when added together, combine to zero. Poker is also a zero-sum game. Even when not played for money or some stakes, there be one clear winner in poker. When played with N-players a loss can simply be accounted for as a negative fraction of a win (i.e. -1/2 if there are three players, because two losses and one win add up to 0). When played for stakes, the total money a player earns is equal in magnitude and opposite in sign to the money the other players lose. All the money totaled will always equal the same amount, and the total change in money won between all the players combined will always add up to zero. Therefore poker is an N-player zero-sum game of imperfect information.

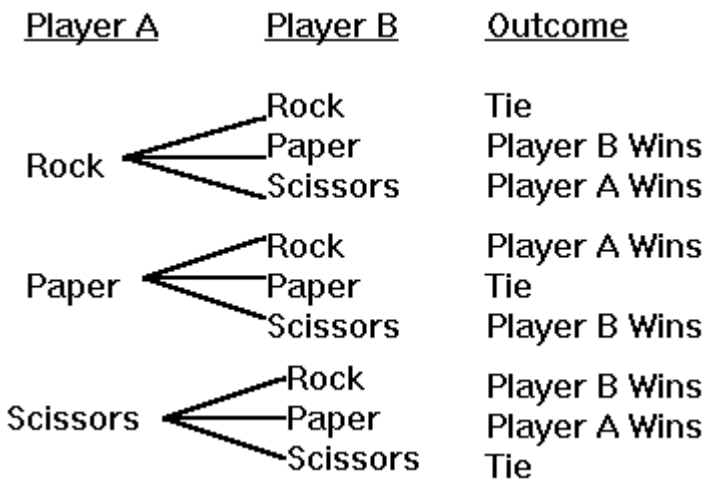
A non-zero-sum game can be more of a real-life application, where negotiation helps two people both succeed. Remember that the definition of "game" in game theory is quite different than we perceive it. Any situation with two or

more people requiring decision making can be classified as a game. Perhaps two people, each independently in business, want to open a chain fast food restaurant in the same town, both of which serve similar items. If they compete, one may strike it rich and the other may go broke. If they work together, they will both do sufficiently but neither will be exceptionally rich. The decisions the two people make can be described in detail by using basic game theory concepts.

The Basic Concepts

Game theory is about choices. In a game such as Advanced Dungeons and Dragons, there may be infinitely many choices. Such a game is not applicable to the laws of game theory. A game such as chess, checkers, or tic-tac-toe can be analyzed by game theory. Game theory is concerned not with games of imagination, but of games of strategy. Just as laws of probability prescribe how games of complete chance operate, laws of game theory apply to games of strategy.

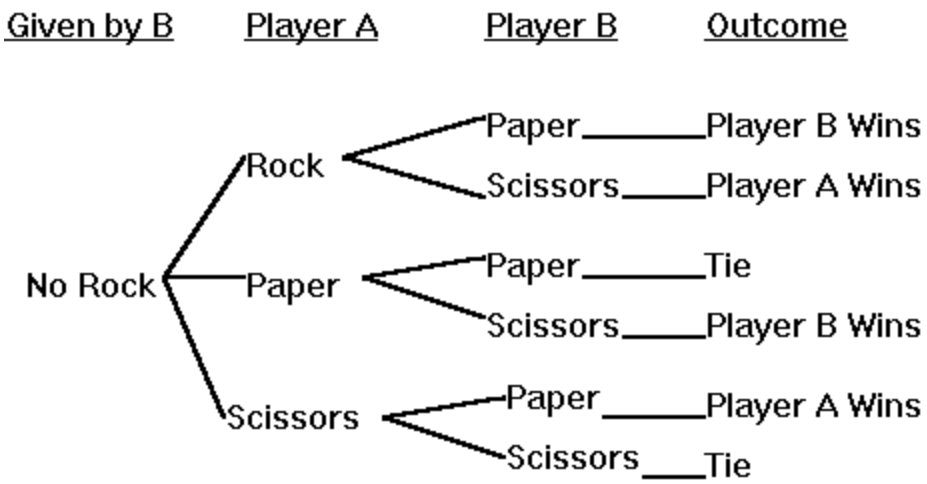
While game theory cannot often determine the best possible strategy, it can determine whether there one exists. Two ideas that can outline games are the game tree and the game matrix. A game tree is a diagram of possible choices that can outline every single possibility within the game. In a game of rock-paper-scissors, there are nine possible outcomes. Player A can choose rock, paper, or scissors. Player B has the same set of choices. Charted as a game tree, this is how the game appears:



Through this analysis, it is clear that any strategy that player A takes will not give him a higher chance of winning this game. This is assumed, of course, that each player makes his choice independent of knowing what the other player has chosen. If player A knew what player B chose, he would be able to win automatically. Because he does not know this information, however, there does not exist a best strategy for either him or player B.

In order to more clearly present the idea of a game tree; allow for the modification of rock-paper-scissors. Let it be assumed that before player A makes his choice, player B tells him one of the two choices he has not made. That is, if player B has chosen "scissors" he may tell player A either that he has not chosen "rock" or that he has not chosen "paper". Given that player B tells player A that he has not chosen "rock", I will analyze player A's options. In this situation, player A could choose scissors and have a one-half chance of winning (a win or a tie). Alternatively, player A could choose rock and also have a one-half chance of winning (a win or a loss). Player A's last option would be to choose paper and have no chance of winning (a tie or loss).

Obviously, player A's best option would then be to choose scissors. Clearly there is a best strategy in this game. The best strategy is to choose the item that would be beaten by the disclosed item that B did not guess (in this case, scissors). Fifty percent of the time player A will win with this strategy, and fifty percent he will tie. Had player B told player A "no paper" instead of "no rock", this strategy would lead to a tie instead of a win as in this situation. Unless he does not use this strategy, player A cannot lose. Now examine this example in game tree form:



By choosing scissors, player A can maximize his winnings in this game. This is a very simple example. To add some complexity, allow the assignment of payoffs. Perhaps player A would be receiving a higher payoff if he chose rock instead of scissors. To compensate for his lower chance of winning, he would need to determine whether the payoffs were fair. Before adding payoffs, a game matrix is a method that needs to be introduced. A matrix is similar to a grid in where all possibilities can be charted. As used in the previous example, here is the game in matrix form:

Player B "no rock"	Player B – Paper	Player B – Scissors
Player A – Rock	-1	1
Player A – Paper	0	-1
Player A – Scissors	1	0

The payoffs are from player A's point of view. A negative score for player A is equal to a positive score for player B. By totaling the columns and finding an average payoff, it is clear that scissors is the best choice for player A. Player B's strategy can also be outlined by this matrix. All columns add up to the same amount, and have the same three quantities. However, it is apparent that player A will never choose paper. With that row deleted, there is a 2x2 matrix remaining. If player B chooses scissors he can either draw or lose. If player B chooses paper he can either win or lose. Therefore paper should be the logical choice for player B. Combined with player A's choice of scissors, it is apparent that player A will win and player B will lose this game.

However, it should seem logical that player A will choose scissors, so the best player B can do is draw by choosing scissors. If player A sees that player B is going to choose scissors because of this, however, he may choose rock and take the win. However, player B can anticipate further and choose paper. Since this psychology can go on forever, we must anticipate the regular odds and stick with them for analytical purposes.

There are a few ways to determine the best choice. While the third column averages 0.5, the second negative 0.5, and the third 0, had the payoffs been different, a different approach might have been better. Another effective strategy is trying to minimize one's loss. That is, you may not have as much to win, but your chances of losing are less. Take this game payoff matrix, for example:

Player B "no rock"	Player B – Paper	Player B – Scissors
Player A – Rock	-10	20

Player A - Paper	0	-5
Player A - Scissors	5	0

In this case, player A still would not want to choose paper because he cannot win anything. However, his choice is now not as clear. With rock, he has a chance of either winning big or losing a lot. With scissors, he can win but the payoff is not great. In minimizing his loss, player A's best choice is A, since he will never lose anything. However, the average payoff for rock is better. Even though he may lose 10, he has the opportunity to win 20. The average payoff for rock is then 5 (both items added and then halved). The average payoff for scissors is only 2.5. If player A only has the chance to play one game, minimizing his loss will be the effective strategy. If player A can play many games, choosing the best average payoff will be more effective.

From player B's standpoint in this game matrix, paper is the choice if he wants to minimize immediate loss. Since player A will never choose paper, player B should never choose scissors since he cannot win anything. Therefore paper is the overall best choice for player B. He will either win 10 or lose 5 depending on player A's decision of either short-term or long-term strategy.

While rock-paper-scissors is a very simple childhood game, it presents a very ideal example for the demonstration of game trees and matrices. Now that these ideas are clear, they can be used to analyze the decision making process necessary in strategy games such as Magic: The Gathering. The effects of probability will also become apparent, but first I will explain the game theory aspect of this game.

Two-Person Game Theory and Magic: The Gathering

Now that the ideas of perfect and imperfect information, zero-sum and non-zero-sum games, game trees and matrices are all clear, I can begin to correlate these ideas with their presence in the game of Magic: The Gathering. While ideas as these are an underlying basis of many games, understanding their applications is helpful in analyzing the roots of what make up the concepts behind the game. Combined with probability analysis this will prove most helpful to understanding the game.

Magic, as a whole, is a zero-sum game. At the end of a duel(1), there is both a winner and a loser, or a tie-game situation. Within the game lie non-zero-sum applications, however. For instance, both players start with 20 life points. The sum of the change in life points is not zero. As a matter of fact, it is far more likely to be negative, since most decks to not utilize cards that gain life. Usually, at the end of a game, one player's life will be zero and the other player's life will be a positive integer. This is not always true, because if both players reach zero simultaneously, the game is a draw. A game can also end because a player has to draw a card for whatever reason, and has no cards left. For the most part, though, reaching zero life is the end of a game, so that will be discussed for all practical reasons.

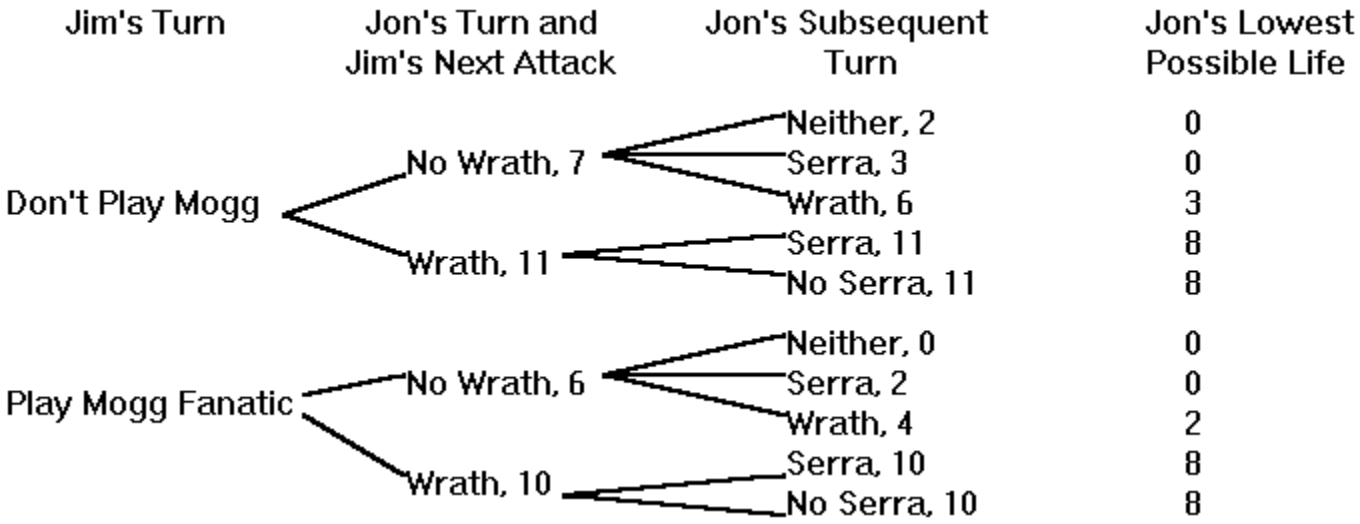
Magic is also a game of imperfect information, with a factor of chance within. You do not know your opponent's cards in hand unless a card which lets you see them is used by either player. The order of the remaining cards in your deck is also an unknown if you are not using cards that let you see this information. The exact contents of the opponent's deck are also an unknown that will not be available for you to see unless you have a card that lets you, such as Jester's Cap(2). Because you know the contents of your own deck, and do not get the same hand every game, there is an inherent factor of chance.

The decisions necessary in Magic can actually be drawn as game trees. Whenever you have to make a decision such as which card to play, whether to wait until later or play a card now, or whether to counter a spell or not, a game tree is involved whether you realize it or not. Allow the following hypothetical situation: Jim has out a Goblin Patrol(3), Mogg Fanatic(4), and Goblin Raider(5). He has three cards in hand: Shock(6), Mountain, and Mogg Fanatic. He has two Mountains in play, both untapped. His opponent, Jon, has out three Plains(7). Jon has no creatures out and five cards in hand. First Jim attacks Jon with the three creatures, dealing five points of damage, bringing Jon down to 12 life. The

questions are, should Jim play the Mountain or hold onto it? Should Jim play the Mogg? Should Jim play the Shock? While I will not yet be analyzing the probabilities influencing the decision, I will analyze the several possible outcomes.

Jim knows that Jon is playing a solid white deck. That is, white is the only color of any of the colored cards in the deck. There are no cards of any other color besides colorless cards. That being said, there are many white cards that are capable of destroying what Jim has in play. Armageddon(8) and Wrath of God(9) are two of these cards. Also, Jon has Serra Angel(10) in his deck to block oncoming attackers with. Wrath of God will destroy all of Jim's creatures. With three lands out, Jon can possibly be holding a Wrath of God in his hand, along with the land he needs to cast it. Therefore, it may not be such a good idea for Jim to play any more creatures. Seeing that Jon is at 12 life, attacking twice more and casting the Shock will let Jim win. If he plays any more creatures, Jim will still have to attack twice to deal sufficient damage, yet will put Jon on the offensive.

However, if he waits too long, Jon may be able to take control of the board. If Jon can play a Serra Angel, he will be able to block one of Jim's attackers. Therefore having another creature to attack with might be a good idea. Mogg Fanatic can be sacrificed to deal one damage, so playing it is never too risky. Being able to attack with it, too, can be a benefit, nonetheless. If he plays the Mountain, he will not gain a sufficient advantage, and may leave himself susceptible to a mid-game Armageddon. If he plays the Shock now, he may give up the opportunity to use it on a creature Jon plays, or may just eliminate the surprise value. However, Jon may be pretty sure that Jim has one in his hand so that may not be an issue. These three decisions have many factors affecting them, as now demonstrated. Here is a general summary of Jim's options and Jon's resulting life total:



Therefore, if Jon plays a Wrath of God next turn he can have a shot at winning the game. If he does not, he will need one next turn, as having a blocker alone will not save him. Since Jim has a Shock in his hand, he can use it to do 2 damage to Jon. Combined with the two Mogg Fanatics, he can beat Jon simply by getting him to 4 life or less. However, if Jon plays the Wrath, Jim will have to sacrifice his Moggs immediately. Playing the Shock ends up not making a difference because to destroy the Serra Jim will need both the Shock and another creature or two. Therefore the efforts are better spent on reducing Jim's life total. Subsequently, if Jon is able to get out a Serra after casting a Wrath, Jim cannot do anything about it except hope to draw something good.

With the resources he currently has he cannot do anything. Playing the Mountain, however, does not seem worth it, since if Jon casts Armageddon, Jim will need to hope to draw another Mountain as well as the additional cards he already needs. Thinking ahead, Jim realizes the best plan is to play the Mogg Fanatic. Saving it will do no good since he already has enough attackers to beat Jon regardless of the Serra, provided he does not play a Wrath. By using this all-or-nothing strategy, Jim is assuming that Jon does not have a Wrath to cast next turn. If he does, the game will be grim for Jim. If he has one the turn after, that is where the only difference comes in.

By saving the Mogg, the advantage gained is that Jim will be able to play it after Jon casts the Wrath. However, since it is so late in the game, Jon will probably be ready to play his Serra the turn after, or something else that can control the Mogg. Hence it is better if he plays it and gets in an extra attack with the Mogg. Being able to put Jon down to 2 life, as opposed to 3 life, will help. If Jim draws another Shock he can win the game right then. Without the extra damage, he would need two direct damage(11) cards. Using an all-out aggressive strategy is therefore the best thing Jim can do in hopes of winning this game.

In order to demonstrate the concept of a game matrix in Magic: The Gathering, I will use a different example. Note that in the case of Magic, both players almost never make game decisions at the same time. The decision making process often alternates from one player to the other. However, the topic of deck selection has several game matrix applications. In Magic, any given kind of deck has both strengths and weaknesses. Deck A may win against deck B most of the time but lose to deck C just as often. Being as there are several types of strategies and sub-strategies within Magic, the choice of deck one uses may often be a great key in determining how he fares at a tournament. While play skill is important, deck match-ups and luck of the draw are factors of the game, also.

Some decks try to play slowly and take control of the game and then win, while others play aggressively and try to win immediately before the opponent can control the game. Jim has a fast red deck with fast creatures and direct damage, a fast green deck with many powerful low casting-cost creatures, and a slow blue control deck. Jon has a fast black creature deck, a white control deck, and a red control deck. For the sake of discussion, let it be given that Jim and Jon tested these decks against each other extensively and found how often their own decks won. Win percentages are from Jon's point of view.

Jon's Win %	Jon Red Control	Jon White Control	Jon Fast Black
Jim Blue Control	70%	20%	80%
Jim Fast Red	50%	40%	20%
Jim Fast Green	30%	30%	50%

The game matrix given by Jim's decks versus Jon's decks can be solved in order for each player to determine which of his own decks is the best one to play against the other player. The simplest method would be to average the possible outcomes and find the result. By doing that, Jon's slow red deck wins 50% of the time, Jon's slow white deck wins 30% of the time, and Jon's fast black deck wins 50% of the time. Using 100 minus the number given since they are from Jon's point of view, Jim's slow blue deck wins 43% of the time, Jim's fast red deck wins 63% of the time, and Jim's fast green deck wins 63% of the time.

If each player uses one of his best overall decks, Jon will either be playing slow red or fast black against Jim's fast red or fast green deck. The simplified game matrix would be a 2x2 which looks like this:

Jon's Win %	Jon Red Control	Jon Fast Black
Jim Fast Red	50%	20%
Jim Fast Green	30%	50%

This being the case, Jon's slow red deck would win 40% of the time, and his fast black deck 35% of the time. Jim's fast red deck would win 65% of the time, and his fast green deck 60% of the time. That being the case, Jon would be more likely

to use his red control deck and Jim more likely to use his fast red deck. Both players would then have a 50-50 chance of winning.

When going to a tournament, however, one must account for every possible deck match-up. That means that instead of just his own decks versus a friend's three decks, he must account for his own decks versus all the different types of decks he will expect to see. Additionally, he must take into account how many of those decks he expects to see. In Magic lingo, this is often referred to as the "metagame". This is the game within a game; determining what you think you are playing against. By determining your chance of being able to defeat a certain deck multiplied by the chance of facing it, and summing up all the products for "match-up probability times win probability" for each opposing deck, one can assess the overall win probability for any given deck. By doing this for each deck and finding which one has the highest probability of winning, a player guarantees himself the highest level of success.

Footnotes:
A duel refers to a single game, while a match is the best two out of three games.
Jester's Cap: Artifact, 4. Pay 2 mana, tap and sacrifice Jester's Cap to look through the opponent's deck and remove any three cards from the game.
Goblin Patrol: Creature - Goblin, R. Echo. 2/1.
Mogg Fanatic: Creature - Goblin, R. Sacrifice to deal 1 point of damage to a creature or player. 1/1
Goblin Patrol: Creature - Goblin, 1R. Cannot be used to block. 2/2
Shock: Instant, R. Target player or creature takes 2 damage. (Yes, this card is obviously a weakened Lightning Bolt, but when building decks out of the in-print pool of cards, only Shock can be used).
Plains: Land. Tap for one white mana.
Armageddon: Sorcery, 3W. Destroy all lands.
Wrath of God: Sorcery, 2WW. Destroy all creatures. They cannot be regenerated (special ability that prevents them from being destroyed).
Serra Angel: Creature, 3WW. Does not tap when attacking. 4/4, Flying.
As opposed to creatures that have to go unblocked to do damage, cards which can do damage directly to a player immediately are often referred to as Direct Damage.

Probability

Probability and Magic: The Gathering

Decision-making is a vital aspect of Magic; however, it is not the sole discerning backbone of the game. To be able to make the right decisions means to know how to determine what is the right decision. Knowing the chance that any given event will occur is what will let the player determine what his proper strategy should be. If your opponent has five cards in hand, four Islands(1) on the table, and nothing else, how do you react? Knowledge of any possible cards in his deck can help.

More importantly, knowledge of cards that are definitely in his deck is even better. If you have a vital spell you want to cast without fear of it being countered, what will your strategy be? Will it change if you know that there are four Counterspells in his discard pile? What if there are two Counterspells and three Forbids in his discard pile? Trying to determine how much countermagic he will have is a difficult task. Nonetheless, it represents a very important situation that comes up often in the game. Knowing how to play your cards is often the key to victory.

Once again, trying to determine the best strategy is where the gray areas lie. Perhaps there is no best strategy, and there are only two equally qualitative strategies. In a case such as that, only chance will determine which strategy is the right one. Your opponent is Zev, and he has two islands untapped, one card in hand, and is at 3 life. You want to win immediately, with the Lightning Bolt in your hand that will do 3 damage to Zev. If Zev has three Counterspells in his discard pile, and there are four total in his deck, should you cast the Lightning Bolt? Assuming he has no other

countermagic in his deck, you want to know the possibility that the one card he is holding is a Counterspell. Before trying to determine the probability that it is a Counterspell, we first need a clear definition of what probability is.

Definition: The probability that an event will occur is equal to the number of favorable outcomes divided by the total number of possible outcomes.

First you have to count up the cards left in Zev's deck. Upon counting, you find that there are 24 cards left. Combined with the card in his hand, there are 25 cards in his deck that you have not yet seen. One of them you know is a Counterspell. The number of unfavorable outcomes in this case is 1: Zev having the Counterspell in his hand. The number of favorable outcomes is 24: The other cards in his deck. The total number of outcomes is 25. Divide 1 by 25 and you get .04, which is 4%. Zev has a 4% chance of having a Counterspell, and therefore you have a 96% chance of winning the game immediately.

What if you had seen the top five cards of Zev's deck? If you had a card that allowed this, and you saw no Counterspell in those cards, your chances would diminish of being able to a Lightning Bolt without it being countered. The number of possible outcomes would be 20. The chance of Zev having a Counterspell would be 1/20, or 5%. Now your chance of winning immediately goes down to 95%. It is not a big drop, but it all adds up. This is just something to consider when playing the game.

However, this question can obviously become more complex. What if Zev had two Counterspells left in his deck? Suddenly this definition of probability is not as useful, since we do not know the number of favorable outcomes. Hypergeometric distribution comes into play here, and is used to determine the answer to such a problem. Drawing cards out of a deck is a major probability situation in Magic; however, there are other players in the game of probability. How effective is shuffling a deck? Can a deck truly be randomized? If so, how many shuffles are necessary to completely randomize a deck?

All of these topics play a large role in the game of Magic. One more major topic, though, is the big question of chance versus skill. When playing in a tournament, you decide to use a fast red deck. You lose all three rounds and go home complaining about how you got paired against decks designed to beat you. How accurate is that statement? To find out if the decks always beat you, first you have to determine whether you played your own deck correctly. Perhaps you played your deck incorrectly and your adversaries played flawlessly. If that was the case, then you were no match for them.

Using the people I have available, I will attempt to draw conclusions on sample deck types about how much of a chance one deck has for winning against another. By then reversing who is playing each deck, new data makes itself present, since each player plays a deck differently. The player who makes the best overall decisions is the one who will win the most with a given deck. By applying that theory, I will be able to chart the statistics of decks and players using this "chance versus skill" method.

Footnotes:
Island: Land. Tap for one blue mana.

Shuffling and Randomization

Since Magic is supposed to be played with a randomized deck of sixty or more cards, the issue of shuffling presents itself at the beginning of the game, as well as some situations within the game that require shuffling. What constitutes fully randomizing a deck? The three most common shuffling techniques used by players are riffle shuffles, pile shuffles, and overhand shuffles. A riffle shuffle is when the deck is divided into two halves and riffled together, alternating one card from each. A pile shuffle is when the deck is dealt out into a number of piles and then grouped back up. An overhand shuffle is just taking part of the deck and moving it to another part of the deck, such as taking a few cards from the middle and moving them to the top or bottom, over and over.

Obviously the overhand shuffle is the least effective shuffle, as "clumps" of cards (that is, small groups of cards) tend to stay together more often and not get randomized in this method. The riffle shuffle and pile shuffles are indeed the methods that professional players use. Simply put, these methods randomize the deck to a much higher degree. Pile shuffles vary among players. Some like to use four piles, while some use seven. I have seen higher-end players use seven piles. This is effective because clumps are nonexistent in this method. Any group of seven cards becomes completely divided using this method. Combined with a few riffle shuffles, this randomizes a deck to a great degree.

How good is a riffle shuffle, though? To randomize a deck of 52 playing cards, it only takes 7 times(1). With 60 cards, it should therefore take approximately 8 shuffles. When many players play a game I often see them overhand shuffle once or twice, and feign a riffle shuffle. These players then draw land after land and wonder why! According to the same web site, it would take 2500 overhand shuffles in order to get the same randomization as 7 riffle shuffles! Again my analysis holds true: Riffle shuffling is a supremely effective shuffling method. When playing a game, the best thing to do is probably pile shuffle with seven piles first, in order to mix up your deck from the previous game. After that, three or four riffle shuffles are incredibly effective. Seven or eight riffle shuffles before each game can be incredibly tedious, but the existence complete randomization is still shaky. Therefore coming close to complete randomization is the best one can do.

Even though pile shuffles and riffle shuffles combine to create a good shuffling technique, there will always be the chance of drawing no land cards or all land cards. However, that is part of being randomized, and will happen every so often. Because of this, the DCI created the Mulligan rule. That lets players shuffle their initial hand into their deck at the very beginning of the game if they are very unpleased with it. The penalty is that their new hand consists of one fewer card. This is a great idea, because the chance of drawing no land twice in a row is very slim. In the next section, I will analyze such probabilities in depth using hypergeometric distribution techniques.

Footnotes:
http://www.scc.ms.unimelb.edu.au/discday/dyk/shuff.html

Drawing Cards From a Deck

Now the question arises again: If you have four of a card in your sixty-card deck, what is the chance you will draw one in your opening hand? If you had only one copy of the card, the problem becomes simple. Probability is defined as the number of chances that the event will occur divided by the total number of possible outcomes. Therefore if you had one copy of a card in your sixty-card deck, you would have a 7/60 chance of drawing it in your initial seven-card hand. If you have four copies of the card, though, does the chance become 4/60? Or perhaps it is more or less than that quantity?

First let me do a quick review of some math concepts. Factorials, permutations, combinations, and hypergeometric distributions are much of what make up this topic. Before I start referring to these concepts I will first explain each of them so the non-mathematically motivated reader can catch up.

A factorial is calculated by multiplying all the whole numbers from 1 up to that number together. It is written in the form "N!" where N is a positive number. For example, 1 x 2 x 3 x 4 x 5 = 120 = 5! which is read as "5 factorial". Zero factorial is defined as 1. While it does not immediately make sense mathematically speaking, trying to find the number of permutations available for zero items makes sense; there is only one way to arrange zero items.

A permutation represents the number of different ways one can arrange N objects. If you have 5 books and want to see how many different ways you can stack them, you would use a factorial. The first book would have 5 possible positions; the next one would have 4 possible positions left, and so on. Therefore there are 120 (5!) ways to arrange 5 books. If you only wanted to arrange 3 of the 5 books, you would only need to compute 5 x 4 x 3 which is equal to 60. That is because you are only using 3 positions.

A combination simply represents a number of possible subsets, and is not concerned with the arrangement of these items. If you were to choose 3 out of 5 books, how many different combinations are possible? For this, you would take

the number of permutations, but then divide by the number of repeated combinations. You do this by taking the factorial of the number of items. For this example, you would find 5 * 4 * 3 and divide it by 1 * 2 * 3. Therefore 60 divided by 6 is 10, and there are 10 different ways to pick 3 out of 5 books.

A hypergeometric distribution is something much more complex. It is used to determine the probability of certain sets of occurrences when extracting elements without replacement. That definition certainly applies to drawing cards from a deck, since you are taking them out. Hypergeometric distribution may seem like an unfamiliar phrase, but it is a concept that we are all fairly familiar with. When drawing cards from a deck without putting them back, this concept applies. This formula can be used to determine how often you draw certain cards from a deck of cards.

The formula syntax is a bit complex, though. Recall the formula for the number of combinations for 3 out of 5 items, that is C (5, 3) is (5 * 4 * 3 / 1 * 2 * 3) or 10. Alternatively, this can be written as 5! / (3! * (5-3)!). This can be converted into the general notation X! / (Y! * (X-Y)!). X is the total number of items to choose from, and Y is the number being chosen. Using the same X and Y notation, the formula for hypergeometric distribution

H (X, Y) is as follows:

H (X1… Xn, Y1… Yn) =

C (X1, Y1) * … * C (Xn, Yn) / C (X1 + … + Xn, Y1 + … + Yn)

However, this can be greatly simplified instead of having to go through each item from 1 to N, where N is the total number of cards. In a two-set case, that is, all the cards you are concerned about being one case and the rest being the other, this is the simplified formula:

H (n) = C (X, n) * C (Y - X, Z - n) / C (Y, Z)

X stands for the number of a certain card that you have in the deck.

Y is the number of cards in the deck.

Z is the number of cards you are drawing.

N is the number you are checking for.

Instead of doing all this arithmetic by hand or with a super calculator that can handle such large factorials, a spreadsheet such as Excel can be used to find hypergeometric distributions. The syntax is HYPGEOMDIST (N, Z, X, Y). For instance, if you have a 60-card deck, what will be your chances of not drawing one of your 4 Lightning Bolts on turn 1? By using HYPGEOMDIST (0, 7, 4, 60) you will get the chance for not drawing the card. Therefore if you want to check for the chances of drawing a Lightning Bolt, you would subtract the result from 1.

Turn 1	60.05 %
Turn 2	55.52%
Turn 3	51.25%
Turn 4	47.23%

Turn 5	43.45%
Turn 6	39.90%
Turn 7	36.58%
Turn 8	33.46%

Turn 9	30.55%
Turn 10	27.84%

Those are the chances of not drawing a Lightning Bolt. On turn 1, you will have 7 cards, and there is a 60.05% chance that you have not drawn one of your 4 Lightning Bolts out of 60 cards. By turn 10 this chance diminishes to 27.84%. Likewise, the chance of drawing one or more by turn 1 is 39.95%, and the chance increases to 72.16% by turn 10.

Hypergeometric distribution has other useful applications within the game. Calculating how many land cards to use in a deck is the base of deckbuilding, as one needs land in order to play his cards. Too many land cards will cause you to draw not enough good cards late in the game, and too few will cause you to stall, giving your opponent the advantage. Some decks will want more land and be able to take advantage of it, and others will want fewer because of smaller mana requirements. If you want to draw four land by turn four often, but not too often, this formula is helpful.

The best thing to do within the game would be to have a 70-80% chance of this happening, then use cards that allow you to look through your deck to get more. Right now I will examine the chances of drawing four land cards by turn four if there are varying amounts in the deck. Using HYPGEOMDIST on Excel, this takes a few steps. For each number of land in the deck (I'll use 16 through 30), you have to determine the chance of drawing 0, 1, 2 and 3 land in 10 cards out of a 60-card deck. Then they have to be added up, and the total subtracted from 1. The final numbers are the chances of drawing 4 land by turn 4.

16	24.99%
17	29.52%
18	34.25%
19	39.12%
20	44.05%
21	48.98%

22	53.85%
23	58.60%
24	63.18%
25	67.54%
26	71.64%

27	75.46%
28	78.97%
29	82.17%
30	85.05%

Depending on what ratio you are willing to work with, it would appear that 28 land seems to give a fair chance of drawing the lands this deck needs. Depending the number of cards in the deck that allow the player to look through extra cards, anywhere from 24 to 28 land is the number to use. Cards such as Impulse(1) help remedy this cause. As long as the player has 2 land to cast it with, Impulse can get the player what he needs anytime in the game.

Footnotes:
Impulse: Instant, 1U. Look at the top 4 cards of your deck. Put one in your hand and the other 3 on the bottom of your deck in any order.

The Average Game

In a game of Magic, there is a lot of decision making and probability. I have already given a glimpse of the decision-making aspect. Now I shall examine the probability aspect. Several random events happen in Magic. The cards you draw are somewhat a factor of randomness. There may be games that you get the same cards, but rarely in the same order. Often times the game might play almost identical to another game because of deck searching cards used to get certain cards, but two games are rarely identical to one another. Deck randomization is one major factor in probability affecting the game. If you need to draw a game-altering card in order to seize advantage of the game, you have to determine the

chance of drawing that card and plan ahead for another strategy should you not draw the card you need. Having a backup plan is fail-safe, simply because relying on a single strategy that will not always work will not get you very far.

There are a few cards in Magic that make you flip a coin, but it is rarely an issue within the game. A couple of years ago there was a coin-flip card that saw plenty of tournament play called Frenetic Efreet(1). It utilized a game mechanic called phasing. When a card phases out it is removed from the game. It phases back in during its controller's next untap phase. This card was good because if it was the target of a card such as Lightning Bolt or Shock, its controller could attempt to save it by phasing it out. It would have a 50 percent chance of being lost as opposed to a 100 percent chance of being lost if it lacked its special ability. Over time, this ability would net its controller an extreme advantage. If the opponent had to use two cards in order to remove a single card, that meant that the player who only lost one card had more resources left. Such card advantage is crucial to gaining an advantage in the game.

Cards like Wrath of God, which can make the opponent lose more than one card, are powerful. When playing more than one creature against a deck with Wrath of God in it, you need to try and figure out whether it is better to play more creatures and try to defeat the person fast, or to anticipate a Wrath and play creatures after. Figuring the probability of a Wrath is useful. Just knowing some basic figures, such as the chance of the opponent drawing one after 5, 6, 7 turns is all a player should be comfortable. Doing hypergeometric distributions on the spot is definitely not something a player needs. Common sense usually dictates the right play.

If there is one card in a deck that can give the deck a severe advantage against an opposing deck, having drawn that card at the right point in the course of a game also adds to randomness. If the card is needed early, getting it within the first three or four turns will often be a deciding factor. This chance is around fifty percent, given there are four of the card in a deck, as shown through hypergeometric distribution.

The largest random factor in Magic is by far randomness of the deck. The cards a player draws from his deck are chosen at random. Therefore what card a player is going to draw is an unknown until he draws his next card. Not having this knowledge limits a player's ability to think and plan ahead. Compare this to the popular game of Tetris. In Tetris, the object is to stack falling blocks so they arrange into lines without gaps. By viewing the next piece, the player easily plans a strategy, and plans ahead in order to succeed. In Magic, the player does not know his next card, and can only plan in the present. That is, unless the player has a card in play which allows him to view the next card or cards in his deck. Generally, being able to plan ahead is limited to the cards in one's hand. Knowing the chances of what cards will be drawn later in is only sketchy, because the player cannot determine exactly when the cards will be drawn.

Take the following situation. You are in a duel with Andrew and you are at 8 life. Andrew is playing a fast black deck with lots of small creatures that are slowly diminishing your life total. You are playing a blue and white control deck and have altered your deck between games (this is called sideboarding(2)) in order to help you defeat Andrew's deck. He has not put in any cards from his sideboard. In order to guarantee your victory against Andrew, you have put 3 copies of the card Light of Day(3) in your deck.

Since you know the entire contents of Andrew's deck, you know that Light of Day will stop him cold in his tracks and let you do whatever you want, since his one method of victory will be gone. It is currently the end of your seventh turn. You played first this game. You still have not drawn a Light of Day, but have all the land you need for it: 4 Islands and 3 Plains. However, if you draw one of your 4 copies of Intuition(4) you will be able to search your deck for a Light of Day and play it immediately. What are the chances of you winning next turn? What are your chances of winning the turn after? What do you do in the meantime if you do not draw a Light of Day?

Since you have taken 7 turns and played first, and we will assume you have used no deck manipulation cards, you have gone through 13 cards. Of those 13 cards, none were Intuitions or Light of Days. Since there are 4 Intuitions and 3 Light of Days, you have 7 favorable outcomes out of 47 cards that can be drawn from next turn. For the turn after, you will have 7 out of 46 if you do not draw a key card next turn. That does not mean a chance of 7/46, though. Using the formula from the last chapter, you would find HYPGEOMDIST (1, 1, 7, 47) because you are checking for 1 card, you are drawing 1 card, you have 7 favorable outcomes, and 47 total outcomes possible. Your chance of winning next turn is

14.9%. To find the chance of drawing the card the turn after if it is not drawn next turn, change the 47 with 46 since there would be one other card drawn next turn which is not directly useful.

To find the total chance between the two next turns, you would need to find the sum of HYPGEOMDIST (1, 2, 7, 47) and HYPGEOMDIST (2, 2, 7, 47) since you are drawing 2 cards, and there can be either 1 or 2 of your 7 key cards drawn. Your chance of winning in two turns is 27.8% if you do not win next turn. What is your chance of winning in 3 turns from now if you do not win within two turns?

By adding up the chances of drawing 1, 2, or 3 copies of your key cards by then, the answer comes to 39.0%. Of course, that is the chance in advance. If after two turns you have not drawn any of the key cards, your chance is actually lower. The same applies to the previous example for winning in two turns, after drawing the first card. Then, the chance is similar to the first example, with one card being drawn. If you do not win after one turn, your chance of winning on the second turn is 15.2%. If you do not win on the second turn, your chance of winning on the third turn increases to 15.5%. This is found by taking the hypergeometric distribution of 46 and 45 cards, respectively, instead of 47.

If you find that these chances are too low, you will need to plan an alternate strategy. Such a strategy might include using your available resources to stalling out Andrew instead of saving them to win the game immediately after you get your lock. For example, you might play a creature now and use it to block one of his in the meantime in order to save you some damage. There is no clear method of probability making the right decision. Instead, a player needs to use probability to plan a strategy. A strategy will provide decisions for any given situation.

Footnotes:
Frenetic Efreet: Creature, 1RU. 0: Flip a coin. If you win, it phases out. Otherwise it is buried. Flying. 2/1.
A word about sideboards: In a tournament, players play the best two out of three games. Between games 1 and 2, and between games 2 and 3, each player can exchange on a one-for-one basis cards between his deck and 15-card sideboard. Players are allowed to use a sideboard optionally; either a 0 card or 15-card sideboard is used.
Light of Day: Enchantment, 3W. Black creatures cannot attack or block.
Intuition: Instant, 2U. Search your deck for any three cards and show them to your opponent, who then chooses one card. The chosen card goes in your hand, and the other two in your discard pile.

Chance Versus Skill

In Magic: The Gathering, there are two major factors which contribute to a victory. The deck and the player are these two factors. One player may play a deck to complete perfection, while another may lose horribly with it every game. Sometimes it may be the player's strategies that are flawed, and sometimes it may be the decks that he has to play against. Certain decks, as I will show, have inherent strengths against others, and that gives them a higher overall win percentage. Given that two players of equal skill are using the two given decks, the deck with an inherent strength against the other deck will win more often. Similarly, if two players of differing skill use the same pair of decks, the better player should usually win with either deck if the decks beat each other half of the time. However, there may be no "50% deck." Some may come close but there is no exact deck that fits this description. Instead, there are several different kinds of decks with varying win proportions.

In order to study this theory, I constructed four decks, and brought them over to my friend Jim Trier's house. Jim is a Magic veteran from a couple years past when he used to collect. He no longer collects the cards, but he stays current with the game and plays online with Apprentice. By being able to represent cards in a computer program without actually having to obtain the cards, Apprentice is a wonderful tool. Of course, playing Magic in person is a completely different experience. Nonetheless, we played 12 sets of 4 games each. In each set, we each took a deck and played 4 games in order to determine a fairly rough estimate of the deck's win ratio, given the player. Note that a deck did not play against itself since there was only one copy of each deck. The results are represented in the following game matrix: (The scores are from Jon's perspective).

Chance and Skill	Jim-Blue	Jim-Green	Jim-White	Jim-Red
Jon-Blue	N/A	0%	100%	25%
Jon-Green	100%	N/A	75%	75%
Jon-White	75%	75%	N/A	100%
Jon-Red	75%	0%	0%	N/A

The following statistics can be obtained from this matrix:

Deck	Overall	Jon's Games	Jim's Games
Blue	29%	42%	17%
Green	79%	83%	75%
White	63%	83%	42%
Red	29%	25%	33%

Jon won 58% of the games and Jim won 42% of the games. However, this is only an approximation since the card draws each deck gets vary a lot. A larger sample of games would be necessary in order to get a better approximation, but the number of games required would be cumbersome and uninteresting. This example is good enough to demonstrate the concept of chance versus skill. The green deck appears to be the overall best, with the white deck second overall best.

The idea of chance and skill interacting comes into play when deck match-ups are analyzed. For instance, when Jon played the green deck and Jim played the blue deck, the green deck won all four games. The same thing happened when Jim used the green deck. Therefore it appears that playing skill did not have an effect on this deck match-up. For the white deck against the green deck, however, this is not the case. It appears that the white deck required more skill to play correctly because Jon was able to beat the green deck three games with it, while Jim was only able to beat the green deck one game with it. Similar conjectures apply to white versus blue, and red versus green. Using a game matrix along with common sense ideas, one can determine where skill plays more of a role in the game than chance does. Even by just looking at the total wins, we see how Jon was able to win more with every deck except the red deck.

The larger the difference between Jim and Jon's wins is, the larger the amount of skill involved in playing the deck. Again, allow me to remind the reader that the amount of data used in this experiment only makes for a very crude estimation of chance versus skill. A bad hand will affect this experiment a lot worse than if there were, say, 100 games played in each trial. For instance, it does not make sense that Jim can play Jon's deck better than Jon can. Jon designed the red deck; so therefore, Jim should not be able to play it better in theory. Nonetheless, card draws can affect such an experiment as this in both ways. Still, it is possible that Jim may actually be the better player of this deck. While it is illogical, it is not to be ruled out as a possibility. The results are just not accurate enough to determine an actual result. However, they are good enough to demonstrate the main ideas.

Deckbuilding

Deckbuilding and Magic: The Gathering

Magic is a very intriguing game in the aspect that there are so many possible deck configurations. Within the infinite possibilities of deck construction lie several effective deck archetypes. A deck archetype simply represents a deck type and all its minor variations. A green deck with many small creatures, commonly referred to as a green horde deck, can have several different configurations. This means some may use one card over another, or maybe just use a different number of some cards. The deck type is the same, but the exact configuration may vary from deck to deck. There may not be a "best" version, but instead, several tuned versions of the deck which operate well.

In Magic, there is no best deck. If there were, everyone would play it! There are dominating decks, however. When a deck dominates, people often play a deck that wins against the dominating deck. This is often called a metagame deck. Often the tournament environment turns into a game of rock-paper-scissors where there is a deck designed to beat the metagame deck, but loses to the dominating deck. The DCI tries to eliminate this from happening by banning cards from deck construction, which are too powerful. In a suitable field, there are many possible deck types. Some may be control strategies, some may be creature swarm strategies, and some may be completely combination based. That means that the deck is designed to set up a combination of cards, which when put together, guarantee victory.

Often it is said that swarm beats control, control beats combo, and combo beats swarm. That is just a generalization, and some decks perform better than others do. Determining which deck is best to play in a tournament requires some simple thought. There is no exact way to determine which deck to play unless you know what other people are playing, and what your odds are of beating them. And still, you only get an approximation.

More importantly than choosing the deck is the deck construction aspect of Magic. Selecting which cards to use for a deck means trying to maximize efficiency and effectiveness. Using cards which are overall effective means trying to find a balance between general usefulness and situational usefulness. Combined with proper management of resources, creating a deck that wins is a complex task. That is often why people rely on decks that have already proven themselves.

Which is more effective though, designing your own rogue (original) deck or using a stock (familiar) deck? Most people agree that each has its own inherent advantages. Stock decks have already proven themselves, so one only needs to be able to play the deck well in order to succeed with it. However, not knowing the deck well is a disadvantage. When you build your own deck you know why the cards are there since you put them there. Not only that, but someone you face in a tournament will most likely know the exact contents of your deck if you play a stock deck. If not that, he will know exactly how to beat it, which cards to deal with first, and the like. If you play a deck you made yourself, the person will not know if you have something more threatening coming up. He will have to guess and make tough judgment calls.

Jamie Wakefield is a Pro Tour player who has been renowned for his success with rogue decks. The deck Jamie has been famous for is a green control deck he calls "Secret Force". Using large green creatures and utility, the deck is able to wreck most of the popular decks. When he first made his appearance with this deck, people did not know how to deal with it since they had never seen it before. The result was that since his deck was at least equally effective, but not known to the opponent, he won. Jamie knew what was in his opponents' decks. However, that same information they did not know about him. The surprise factor won him many games, and qualified him for Pro Tour: New York.

Usually, though, rogue decks have limited success. Still, the respect factor is great for those who can design new decks, play them well, and win games. It is easier for someone to take a deck already in existence and play it to perfection than for the same person to design a deck and win just as much. The reward in the latter case is that the person can truly call the deck his own.

Deckbuilding: Deck Archetypes and Deck Strategy

While it is impossible to classify every deck into a concrete category, the vast majority of tournament-level decks fall into one of three categories. There are control decks, aggressive decks, and combo decks. While it is not a definite rock-paper-scissors scenario, aggressive decks tend to beat control decks more often than not. However, it really depends on which kinds of decks they are. Control decks can often win against combo decks but only if there is a lot of countermagic in the control deck. Combo decks can often win against aggressive decks simply because they can ignore the pressure and win immediately at a certain point.

The control deck utilizes cards that can neutralize the opponent's threats. Counterspells, creature elimination, and other threat neutralization cards form the backbone of a control deck. Direct damage can be used defensively to destroy creatures, but has the flexibility of being used to damage the opponent if necessary. Blue-red decks were once a popular phenomenon based on this theory. Counterspells could take care of any major threats while direct damage either destroyed opposing creatures or worked at finishing off the opponent directly. Cards that can tap creatures or prevent them from attacking are also useful in a control deck. Threat management is the critical idea here. Being able to deal with opposing threats means preventing the opponent from winning. By being able to prevent the opponent from winning, the control player allows himself to win.

How the control player wins is usually unimportant. A single large creature is usually fair game, while cards such as Millstone(1) are also popular in this kind of strategy, in order to run the opponent out of cards. While damage is the most common method of victory, forcing the opponent to not be able to draw a card is an often-overlooked method of winning. Control decks can be any color, but often use blue for countermagic. White has mass destruction cards such as Armageddon and Wrath of God to destroy lands and creatures, red has direct damage to augment countermagic, and black has discard and more efficient creature removal. Green is not too popular as a control color but has cards such Wall of Blossoms(2) that make for good defense.

Aggressive decks often take the form of a creature swarm strategy. If you can present the opponent with more threats than he can deal with, you will be able to defeat his control strategy. Conversely, though, if the control player can assert himself before the swarm strategy wins, the control player can take victory for himself. Sometimes the game is not over immediately after it seems the control player has his plan working, it can happen that the swarm player uses a sneaky tactic in order to win his victory back. Usually a Wrath of God will put a gaping hole in the creature-based deck's plan by destroying all the creatures he has already played. However, once more creatures hit the table the control player will either have to come up with another Wrath or lots of creature elimination.

Sometimes the aggressive deck takes on other forms. A deck relying mostly or even totally upon direct damage cards can be effective. In this case, creature elimination attempts are futile. A "burn" deck can cast lots of direct damage spells like Shock, Lightning Bolt, and Fireblast(3) with one simple goal: To bring the opponent from 20 to 0 life. With plenty of Mountains in this all red deck, Fireblast makes for a great finisher. In order to beat a burn deck, the control deck has to either use a lot of countermagic, or a lot of life gain.

Another aggressive deck, usually a creature swarm deck, will have a greater chance of winning against a burn deck. Bringing out large amounts of fast creatures can often overwhelm the burn deck, and force the burn player to be defensive. By forcing the aggressive player to play defensively, that puts a large hole in his plans and ruins his strategy. Sometimes, however, the creature swarm deck will get a slow start and the burn deck will win. Anything can happen in the world of Magic.

The third major type of deck is the combo deck. While many players frown upon many kinds of combo decks, this reigns to be one of the most effective strategies in Magic. Over the course of Magic history there have been many famous Magic decks based around combos. While I will not go into detail about them, I will explain the concept. Zvi Mowshowitz, a student at Columbia University, designed a deck in 1998 that he dubbed TurboZvi. The deck brought out a card called Dream Halls(4) very fast, which allowed both players to play spells practically free. The deck drew lots and

lots of cards up until the point where it could either create a lot of mana to win with a large direct damage spell, or make the opponent run out of cards. With a large mana engine, anything is possible.

If Zvi played against a creature swarm deck, he would just ignore the threat and win third or fourth turn, before his opponent had any chance of winning. Since the creature swarm deck offered no threats, he was completely safe from its simplistic strategy. Even still, the deck had some countermagic in it in order to save itself just in case. Against a control deck, though, the deck became a lot tougher to play. If the opponent played his cards right he could easily win.

Dream Halls helped the countermagic user more than Zvi. The deck was okay but not as good as other combo decks such as Prosperous Bloom, named by two of the main cards in it, Prosperity(5) and Cadaverous Bloom(6). This deck was very famous, and brought Mike Long victory at Pro Tour: Paris. By generating a large amount of mana with a card combination that just about works itself together very easily, this deck was powerful in its day. Of course, there were ways to beat it, by using effective enchantment removal and countermagic. Mike knew how to play his deck to perfection and took on the competition his best.

While combo decks, creature decks, and control decks are the three main types of decks which have presented themselves, there are many decks which fall into more than one category, and even some which fall into none of the categories. However, it is safe to generalize that the vast majority of decks are one of these types. I have played many decks and I see that these kinds of decks stand out about all others. How does a given deck perform? That is the next question I will attempt to answer.

Footnotes:
Millstone: Artifact, 2. Pay 2 and tap to take the top 2 cards from any player's deck and put them in his discard pile.
Wall of Blossoms: Creature - Wall, 1G. When Wall of Blossoms comes into play, draw a card. 0/4. (Walls cannot attack).
Fireblast: Instant, 4RR. Does 4 damage to a creature or player. Caster can sacrifice two Mountains instead of paying the casting cost.
Dream Halls: Enchantment, 3UU. Instead of paying the casting cost of a spell, any player may choose to discard a card that shares one or more color with the spell.
Prosperity: Sorcery, XU. Each player draws X cards.
Cadaverous Bloom: Enchantment, 3BG. Remove a card in your hand from the game to get 2 green or black mana.

Deckbuilding: Deck Comparison and Winning Ratios

A good deck needs to be able to not only have a good chance of winning overall, but it also needs to have a good chance of winning against the most popular decks. If "Draw-Go" (a popular blue control deck) is the dominant deck, playing a combination deck such as Prosperous Bloom is usually a bad idea. Even if Bloom may win about 75% of the time against all the other decks, it is no good if it cannot beat the most popular deck consistently. Of course, no one deck is usually ever dominant.

To date, only twice was there a deck that truly dominated the tournament scene. In November-December of 1998, after the release of the Urza's Saga standalone expansion, a completely overpowered combo deck based around a card called Tolarian Academy(1) emerged. This was by far the best deck, beating everything else hands down. In the summer of 1996, a deck based around Necropotence(2) gave the card popularity, and the combined elements of total resource destruction made this deck dominant. Mainly, a dominant situation only existed with these two decks. Otherwise, there have been popular decks, but never a dominating deck quite the way these decks dominated. That being said, what is the best way to see how effective your deck is against a field of other decks?

The first obvious step is to know how to play your deck as effective as possible. By playing the deck against various other decks, you need to learn every deck's strengths and weaknesses. Once you know that, you can capitalize on these areas and play your deck to maximum effectiveness. After you've done all your playtesting, you need to figure out the most

popular decks and make sure your deck (and/or sideboard) can handle them. If you keep losing to the most popular decks, you cannot possibly win a tournament.

For this purpose, I played my red control deck many games in order to demonstrate whether playing it in a tournament would be a good idea. I came to the conclusion that it is a fairly good deck, though it can by no means handle the competition hands down. The players I played against had different decks. Because of this, the deck's performance is better analyzed by viewing the particular decks I played against. After playing thirty games against various players with various decks, this is how I fared against all the players, and the most common decks:

Vincent	Matty	Mike	Robert	Jay	Willie	Andre	Thiago
3-0	2-0	2-0	3-2	3-2	4-5	0-2	0-2
100%	100%	100%	60%	50%	44%	0%	0%

(A = Aggressive Deck, C = Control Deck, M = Miscellaneous, O = Combo Deck)

Maiden-C	Necro-C	Black-A	Stompy-A	Death-C	Sligh-A	Andre-M
2-0	3-2	3-2	3-3	1-2	0-3	0-2
100%	60%	60%	50%	33%	0%	0%

Here are brief explanations for these decks: The first deck was a blue deck based around the opponent having lots of cards in hand through various methods and taking damage from Iron Maiden(3) because of it. The second deck was a modern version of the Necropotence deck, designed around destroying the opponent's hand and creatures, gaining life and drawing cards. The third deck was a very fast black weenie swarm deck. The fourth deck was an even faster green weenie swarm deck nicknamed "Stompy". The fifth deck was a deck called "Death" with many different creatures, Survival of the Fittest(4) and Living Death(5). That lets the player stuff his discard pile full of creatures then bring them into play.

The sixth deck was Sligh. Sligh is an aggressive red weenie swarm and direct damage deck. The name originated from a man named Paul Sligh who created a red control deck during the summer of 1996. The deck evolved into the mega-aggressive modern day weenie swarm deck, but the name stayed. The last deck was Andre's own creation: a black and white deck based around Remembrance(6), Phyrexian Reclamation(7) and many effective creatures. This proved to be very effective for him.

My overall win ratio was 17 to 13. The fact that I won 57% of the games I played is nice to know, but not too useful. How I performed against the average deck is more useful. By averaging how I did against each deck and putting it together, I would have a rough estimate of how I would do at a tournament against an average field. The result is that I win 43% against the average deck. This does not take into account the sub-standard decks I play against every now and then, as I would never expect to play against anything like that in a tournament. A player with little experience is not likely to pay the admission fee for a tournament, when he is more likely to just play for fun. That being said, it is not a good idea to

include those wins, as inflating my win ratio does not help me as a player. Even still, a much larger number of games are necessary to get a better approximation for my true win percentage.

This procedure assumes there is an even field, however. Perhaps I would end up having a higher chance of winning if there were more decks in the tournament that I had a higher chance of winning against! Given a field of 50% Iron Maiden decks, 25% black weenie swarm decks, 15% Sligh, and 10% Death, I would have a higher chance of winning. Of course, Death and Sligh are a lot more popular than Maiden and black swarm. Given two tournament scenes with different percentages of these decks, with my deck being the only red control deck, I will give a sample situation. Recall that finding the approximate chance of winning the tournament consists of multiplying the chance of winning against a deck by the chance of playing the deck, and adding up that total for every deck. I can put this into mathematical notation as a Riemann sum because I am summing up the chance of winning N different decks multiplied by the chance I will play against them:

$$\sum_{K=1}^{N} F(K) * C(K)$$

F(K) represents the amount of the deck in the field

C(K) represents your chance of winning against the given deck

Deck Name	Situation #1 (Good)	Situation #2 (Bad)
Survival/Living Death	10% * 33% = 3.3%	50% * 33% = 16.7%
Red "Sligh"	15% * 0% = 0%	25% * 0% = 0%
Iron Maiden	50% * 100% = 50%	10% * 60% = 6%
Black Speed Weenie	25% * 60% = 15%	15% * 60% = 9%
Total Win Percentage	68.3%	31.7%

Therefore the amount of each deck at the tournament scene can drastically affect my chance of winning the tournament. The Riemann sum is an important idea here, as it is used to sum up all the possibilities into a total chance amount which represents the probability of me winning the hypothetical Magic tournament.

Footnotes:
Tolarian Academy: Legendary Land (there may only be one in play at a time). Tap for one blue mana for every artifact you have in play.
Necropotence: Enchantment, BBB. Skip your draw phase. Pay 1 life: Draw a card during your discard phase.
Iron Maiden: Artifact, 3. All opponents take 1 damage at the end of their upkeep for every card in their hand more than four.
Survival of the Fittest: Enchantment, G. G: Discard a creature to search your deck for any creature, show it to all players, and put it in your hand.
Living Death: Sorcery, 3BB. Switch all creatures in play with those in the discard piles.
Remembrance: Enchantment, 3W. Whenever one of your creatures leaves play you can search your deck for another copy and put it in your hand.

Phyrexian Reclamation: Enchantment, B. 1B: Pay 2 life to take a creature from your discard pile and put it in your hand.

Deckbuilding: Card Efficiency and Resource Management

When choosing cards for a deck, the player needs to decide which cards will do the best job. That is to say, if I want to use white creature removal, which card is most effective for this purpose? Which countermagic cards will fit into my deck the best without a mana problem? Which creatures will work best in my deck? These questions relate to the idea of card efficiency. Getting the best possible card for the lowest possible cost means the highest efficiency.

In current Magic, many cards that are very efficient are no longer printed because they were so powerful. Swords to Plowshares, which I mentioned earlier, is one of these cards. For one white mana, this card can remove any creature from the game. The life gain simply does not balance out this card. Chances are, you may be playing a control deck and you don't care how many life points your opponent has. This card will simply keep you alive and that is all that matters. Combined with cards like Timetwister(1), this card is even more powerful because you get back your Swords and your opponent's creatures are hopelessly gone for the game. Timetwister was another card which ceased being printing, albeit a lot sooner than the Swords.

When choosing which creatures to use for a deck, if any, the best ones are obviously ones with a higher power and toughness to casting cost ratio. For a completely aggressive deck, only sheer power is of importance. The almighty Ball Lightning(2) boasts a power of six with a casting cost of three. For fast red decks, this has always been a popular card. If the defending player cannot deal with it he will take six damage, which is about one third of his beginning life points!

Combined with direct damage spells like Lightning Bolt which deals three damage for one mana, and Fireblast which can deal four damage for no mana (at the slight cost of losing your own land), fast red decks were popular while those cards were in-print. Winning the game is usually the result of the resource sacrifice from Fireblast. The popularity of the deck was because of the supreme card efficiency of cards like these. Of course, after these cards were taken out of print and replaced by newer sets, this deck was no longer viable.

Usually creatures with a casting cost of one will have a power of one. Sometimes they will have a power of two, but at a cost. It may not always be able to be used to attack, or it may deal damage to its controller. The purest breed of aggressive decks will take blazing speed at any cost. Such are fast black, red, and green decks. Blue and white do not have any extremely efficient creatures, but they do have creatures with good utility. That makes blue and white creature-based decks better as a defensive strategy. Aggressive decks utilize low-cost creatures and damage spells. Defensive decks utilize high-efficiency threat management removal spells. Disenchant(3) is another prime example of a popular removal spell. Just as Swords to Plowshares is to creatures, Disenchant is for artifacts and enchantments. However, creatures are usually a bigger threat in Magic, so Disenchant is a more balanced card. That is why it has been in the current tournament environment since the beginning of the game.

Being able to maximize what resources you have available is what makes your strategy more effective. By being able to remove more than one opposing card with only one of yours, you gain card advantage. Card advantage is not the only kind of advantage, though. By using cards which keep the opponent's resources unavailable (such as Winter Orb(4)), you gain time advantage. This means that since your opponent cannot play his cards as usual, he is delayed and you gain more time to do whatever you have to within your strategy. This also means that the opponent will often need more time to react to your other threats.

Yet another kind of time advantage lies in taking multiple turns. If you can take more turns than the opponent can, you have more time to maximize your strategy's effectiveness. This is a major resource advantage because it allows you to bring more resources into play. By taking another turn you get to attack again, play another land, and draw another card. This is why the card Time Walk(5) was banned from tournament play. For two mana it let the caster take another turn. However, a more balanced version was later printed, called Time Warp(6). For five mana this had the same effect. By delaying when the player can use it, it balanced out its advantage with a high cost.

Resource advantage can be gained either through drawing more cards, removing multiple cards with only one, or by denying the opponent resources. Taking extra turns is another form of denying the opponent resources, in addition to land destruction, hand destruction, and creature destruction. In order to maintain control of the proverbial game board, cards that are able to make the opponent discard, lose land, and lose creatures are a valuable resource in Magic when used effectively. Many decks are based around one or more of these concepts. When well tuned and well played, these decks often lead to great success. Nonetheless, there are many other viable strategies, such as the aggressive one: overwhelming the opponent with many threats. While there may not be a best strategy in Magic, there are several to choose from. How the player uses the strategy is what will lead to his varying degree of success.

Footnotes:
Timetwister: Sorcery, 2U. Each player shuffles his hand, deck, and discard pile together, and draws seven cards. Put Timetwister into your new discard pile.
Ball Lightning: Creature, RRR. Can attack the turn it is brought into play. 6/1, Trample (any damage over the total toughness of the creatures blocking it is dealt to the defending player).
Disenchant: Instant, 1W. Destroy any artifact or enchantment.
Winter Orb: Artifact, 2. Players only get to untap one land each untap phase.
Time Walk: Sorcery, 1U. Take another turn after this one.
Time Warp: Sorcery, 3UU. Take another turn after this one.

The Big Game:

Probability, Statistics, Game Theory, and Magic

The final question arises: How do all these concepts come together as a whole? The various mathematical aspects of Magic do not make up the game individually; they do together as pieces of a puzzle. That being said, there exists a relation between the various mathematical components of the game. Each part of it helps form the framework of the basic situations that a Magic player encounters every game.

Take the following situation. Put yourself in my shoes. The date is February of 1999 and you are trying to qualify for the Junior Super Series. You know that the majority of the decks will be combination decks based around getting a combo out and winning on the third or fourth turn. Using your knowledge of game theory, probability, and statistics, what deck will you play? What will your chances be of winning?

Here is how the situation was: The field was made up of about 40 people. It would be a fair estimation to say that 30 were playing combo decks, based around winning very fast with a card combination. Knowing that most creature-based decks will lose to combo decks, it is also a fair assumption to say that there would not be many creature-based decks. That prediction was true. Therefore, a control deck appeared to be the best deck to have played. I ended up playing a deck with 20 countermagic cards in it, with very little creature removal. That way I could counter most of the combo threats, and present my own three-card instant win combo. It was a control deck that had the capability to with instantly with a combo. The massive amount of countermagic made it very strong against combo decks, since they would have trouble getting their combo out.

I had playtested with Zev Gurwitz over thirty games against his combo deck, which he had already qualified with. I won about sixty percent of the games, simply because his deck was both very fast and had the capability to win against control decks. Given that I could beat the average combo deck 75% of the time, and that 75% of the field would be combo, I had one piece of the puzzle solved. The other 25% of the field would be a random assortment of decks. Some creature swarm decks, some control decks, and some other miscellaneous decks. Some which I could beat almost all of the time, some which I had little chance against. To say I would win 25% against fast decks and 75% against slow decks was a fair assumption. I did not expect to play against any random decks in most of the tournament simply because they would be eliminated very soon. Therefore my chances were about this much:

Deck Type	Chance of Win * Chance of Match-up
Various Combo Deck	75% * 75.0% = 56.3%
Various Speed Deck	25% * 12.5% = 3.1%
Various Control Deck	75% * 12.5% = 9.4%
Total Win Percentage	68.8%

This gave me an extremely high chance of winning. Knowing the metagame, that is, what decks other people were going to be using, was an extremely effective tactic. While I would lose to fast decks, I guaranteed myself a win against the slower decks and especially the combo-reliant decks.

In round 1, I played against a combo deck called "Spiral Blue" and won. In round 2, I played against another Spiral Blue deck and lost, as the deck was played by a pretty good player, and the only other match I had played against this deck was in round 1. In round 3, I played against a fast red deck and lost. That was the small price I paid for playing the metagame. However, I faced two other combo decks in round 4 and 5 and won against them both. The 3-2 record put me into the top 8. I had won 75% against the combo decks (3-1) so my estimate was very good. In the top 8, I played against a control deck and won, and then two combo decks and won against both of them. The last round was, incidentally, the same opponent from round 2. This time around, I got luckier perhaps. Whatever it was, I had won because I knew the chances and played the metagame correctly.

Knowing which deck to play is a definite game theory application. The decision making process formed by game matrices and relative probabilities constitutes a calculation of chance. The statistics that I used in order to formulate my chances of winning against each deck are strikingly similar to those used during my win-loss ratio calculation experiment. Such a technique is very handy when trying to win a major tournament. While I do not win them on a regular basis, I do fairly consistently. Statistics and probability are only an average. They are not a fixed value. Instead, they serve as an approximation. Knowing how to use them is what makes a player more informed.

Math and the Average Player

By now I have clearly demonstrated how math is a big part of Magic. From probability, decision making, and strategic applications, it is apparent that math forms the backbone of what makes the game so intellectual. However, the question arises to whether players actually identify and use the math components. To find out about how much math is actually seen by players of the game, I conducted a poll on The Magic Dojo. I asked players to send me responses to the following questions:

What is your name and age?

How long have you been playing Magic?

What is your background in math (what courses have you taken)?

To what extent to you think math is an important part of strategy and game play?

How, if at all, does math play a role in your game play? Explain.

Do you think someone who is more knowledgeable in math has a higher playing potential than someone who does not does? Why or why not?

This survey was posted on The Magic Dojo (www.thedojo.com) on May 14th. Over the course of May 14 through the morning of June 9, I received 182 responses. They came from players of varying age and experience. Most recognized the mathematical aspects; yet most agreed that math does not make for a better player. Critical thinking skills are the basis

of being a better player. Nonetheless most recognized that math does exist behind the scenes of the game. After all, Richard Garfield, the designer of the game, is a graduate student in mathematics! After reading the responses, I sorted them out in hopes of drawing conclusions. The following are my analysis of the information I gained from this survey:

I. Age distribution

12-15	16-19	20-23	24-27	28-31	32-35	36-39	40-43	44+
18.6%	26.6%	21.5%	12.4%	13.6%	4.0%	1.7%	1.1%	0.6%

The average age of the Magic player is a high school or college level student. There were fewer middle school and graduate level students that answered the survey. There were even fewer older players, though there were some. The high school to college range can be attributed to the targeted interest group. Most males this age are interested in strategy gaming far more than their older and younger counterparts. Note that I did not include gender in this poll. The majority (more than 95%) was males. Females generally do not constitute much of the portion of Magic players, generally because they do not take as great an interest in strategy games. These are just statistics, not an opinion.

II. Experience distribution

0-1 years	1-2 years	2-3 years	3-4 years	4-5 years	5-6 years
5.3%	12.9%	16.4%	25.1%	31.6%	8.8%

Please note that X-Y years denotes greater than X, and less than or equal to Y. Therefore 2-3 years means more than but not including 2 years, up to and including 3 years of experience playing Magic. It appears that the majority of players have been around since when the Revised (3rd) edition had been released. I know that the popularity of the game had actually peaked then, so these results do in fact make sense. Part of the result may be that this only represents the field of Dojo readers, which it does, but still it seems fairly accurate as a general distribution.

III. Background knowledge distribution

Middle School	High School	Calculus	College Level	Graduate Level
3.0%	33.3%	14.9%	38.1%	10.7%

Middle School indicates general middle school math education. High School indicates Sequential Math courses I through III, as well as Precalculus. Similar algebra, geometry and trigonometry courses correspond as well. I chose not to include Calculus under the category of college level math. The math levels seem to correlate directly with the age of the players. Therefore the average player can be a high school or college student well educated in mathematics, and has played Magic for 4 or 5 years. It would appear that an interest in mathematics corresponds with an interest in Magic. This is usually the case. People like Toby Wachter from Great Neck, NY are not as proficient in math and still enjoy the game a great deal.

IV. Recognized concepts, use of math, and view of importance of math

	Very Little	Somewhat	A great deal
Concept recognition	9.0%	36.7%	54.2%
Use of math	18.1%	31.3%	50.6%
Importance of math	20.5%	36.1%	43.4%

It is very apparent that Magic players recognize the mathematical concepts behind Magic, use these concepts in their game play, and see their relative importance. Beginners may only see the subtleties such as life counting, damage dealing, and other addition/subtraction ideas such as creature combat. More advanced players identify mana curves, casting cost analysis, land ratios, and timing issues. Experts see advanced probability, statistics, and metagame analysis. Most players agree that decision making is superior to mathematical ability when it comes to playing the game.

In deckbuilding, mathematical skill comes into play more often, though intuition often reigns supreme. While math is a big part of Magic, it may not necessarily make a better player. Decision-making does, though. Game theory, an economics application, is more useful to the average player then numerical mathematics. Deck creation is where the math comes into play. Game mechanics are full of math. The game play is not as full of math. While beginners only view math as life totals, experts see the big picture. Math thinking skills are useful in the game either way.

Players use a fair amount of math either at a conscious or subconscious level. While it is an underlying part of the game, it does not necessarily enhance one's playing skill to a large degree. Knowing math never hurts, though. In deck construction, math is great. In playing, fast arithmetic is also a good thing. Logic and reasoning skills are also good things to take with you onto the gaming field. Play experience is more valuable than pure mathematical experience, though. All players seem to agree on these ideas.

Personal Applications

I will now attempt to answer my own questions after doing my research and analysis. My name is Jon Prywes, and I am 17. I have been playing Magic for four and a half years. I have taken everything up to BC Calculus and will be taking Linear Algebra in the fall. I see math as a very important part of Magic. Knowing how to create an efficient deck and play it well requires a basic understanding of probability and statistics. Creating decks on your own time can utilize some major mathematical ideas, while in the actual playing area, logic and reasoning are tested very intensely.

I will base my strategy often upon what I anticipate my opponent doing. Sometimes I have to figure out the odds and base my strategy around them. While math is a big part of my game, I don't think that it makes me too much of a better player. It may give me an advantage against someone who does not know as much math, but it is definitely a part of the game itself. Mathematical reasoning skills are very useful, and I would not be able to play the game as well as I do without them. Making decisions at lightning speed is something required of any tournament-level Magic player. I need to be able to do that a lot. One small mistake has meant a match loss too many times. Playing flawlessly with reasonable speed is a key to success. Being able to acquire this skill requires countless hours of practice, sleep, and a proper diet. The mathematics are definitely in there, though.